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ABSTRACT

This paper explains in user-friendly terms why multivariate statistics are so important in educational research. The basic logic of canonical correlation analysis is presented as a simple or bivariate Pearson "r" procedure. It is noted that all statistical tests implicitly involve the calculation of least squares weights, and that all parametric tests can be conducted using canonical analysis, since canonical analysis subsumes parametric methods as special cases. Canonical analysis is potent because it does not require the researcher to discard variance of any of the variables, and because the analysis honors the complexity of a reality in which variables interact simultaneously. Three major classes of procedures for evaluating the importance of specific variables in canonical correlation analysis were explored. Various procedures in each class were illustrated in a concrete fashion using a single small data set for heuristic purposes. Appended program files for the Statistical Package for the Social Sciences and the Statistical Analysis System may facilitate further exploration of the concepts presented. (A 77-item list of references is included. Seventeen data tables, one graph, and outlines of computer programs used are provided.) (Author)

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Variable Importance in Multiple Regression and Canonical Correlation

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ABSTRACT

The paper explains in user-friendly terms why multivariate statistics are so important in educational research. The basic logic of canonical correlation analysis is presented as a simple or bivariate Pearson *r* procedure. It was noted that all statistical tests implicitly involve the calculation of least squares weights, and that all parametric tests can be conducted using canonical analysis, since canonical analysis subsumes parametric methods as special cases. Canonical analysis is potent because it does not require the researcher to discard variance of any of the variables, and because the analysis honors the complexity of a reality in which variables interact simultaneously.

Three major classes of procedures for evaluating the importance of specific variables in canonical correlation analysis were explored. Various procedures in each class were illustrated in a concrete fashion using a single small data set for heuristic purposes. Appended SPSS-X and SAS program files may facilitate further exploration of the concepts presented.

Canonical correlation analysis is a powerful analytic method that may best honor the complex nature of the reality to which the researcher wishes to generalize. As Kerlinger (1973, p. 652) suggests, "some research problems almost demand canonical analysis." Similarly, Cooley and Lohnes (1971, p. 176) suggest that "it is the simplest model that can begin to do justice to this difficult problem of scientific generalization." Wood and Erskine (1976) identified more than 30 applications of these methods. More recently, Thompson (1989a) cited roughly 100 canonical applications reported during the last decade.

Hinkle, Wiersma and Jurs (1979, p. 415) noted that "it is becoming increasingly important for behavioral scientists to understand multivariate procedures even if they do not use them in their own research." And recent empirical studies of research practice do confirm that multivariate methods are employed with some regularity in behavioral research (Elmore & Woehlke, 1988). There are two reasons why multivariate methods are so important in behavioral research, as noted by Fish (1988).

First, <u>multivariate methods limit the inflation of Type I "experimentwise" error rates</u>. The seriousness of "experimentwise" error inflation, and what to do about it, are both matters prompting some disagreement (e.g., Bray & Maxwell, 1982, p. 343, 1985, p. 10; Hummel & Johnston, 1986). But it is clear that, "Whenever multiple statistical tests are carried out in inferential data analysis, there is a potential problem of 'probability pyramiding'" (Huberty & Morris, 1989, p. 306). And as Morrow and

Frankiewicz (1979) emphasize, it is also clear that in some cases inflation of experimentwise error rates can be quite serious.

Most researchers are familiar with "testwise" alpha. But while "testwise" alpha refers to the probability of making a Type I error for a given hypothesis test, "experimentwise" error rate refers to the probability of having made a Type I error anywhere within the study. When only one hypothesis is tested for a given group of people in a study, "experimentwise" error rate will exactly equal the "testwise" error rate.

But when more than one hypothesis is tested in a given study, the two error rates will not be equal. Witte (1985, p. 236) explains the two error rates using an intuitively appealing example involving a coin toss. If the toss of heads is equated with a Type I error, and if a coin is tossed only once, then the probability of a head on the one toss and of at least one head within the set of one toss will both equal 50%. But if the coin is tossed three times, even though the "testwise" probability of a head on each toss is 50%, the "experimentwise" probability that there will be at least one head in the whole set of three flips will be inflated to 87.5%. These dynamics are illustrated in Tables 1 and 2. Researchers control "testwise" error rate by picking small values, usually 0.05, for the "testwise" alpha. "Experimentwise" error rate can be limited by employing multivariate statistics.

INSERT TABLES 1 AND 2 ABOUT HERE.

Paradoxically, although the use of several univariate tests

in a single study can lead to too many null hypotheses being spuriously rejected, as reflected in inflation of "experimentwise" error rate, it is also possible that the failure to employ multivariate methods can lead to a failure to identify statistically significant results which actually exist. Fish (1988) and Maxwell (in press) both provide data sets illustrating this equally disturbing possibility. Thus, "correcting" the testwise alpha level (e.g., with a Bonferroni correction--Huberty, 1987) so as to control experimentwise error rate inflation is **not** a satisfactory solution to this problem. The basis for this paradox is beyond the scope of the present treatment, but involves the second major reason why multivariate statistics are so important.

Multivariate methods are often vital in behavioral research because <u>multivariate methods best honor the reality to which the researcher is purportedly trying to generalize</u>. Since significance testing and error rates may not always be the most important aspect of research practice (Thompson, 1989c), this second reason for employing multivariate statistics is actually the more important of the two grounds for using these methods. Thompson (1986, p. 9) notes that the reality about which most researchers wish to generalize is usually one "in which the researcher cares about multiple outcomes, in which most outcomes have multiple causes, and in which most causes have multiple effects." Tatsuoka's (1973, p. 273) previous remarks remain telling:

> The often-heard argument, "I'm more interested in seeing how each variable, in its own right, affects

> the outcome" overlooks the fact that any variable taken in isolation may affect the criterion differently from the way it will act in the company of other variables. It also overlooks the fact that multivariate analysis--precisely by considering all the variables simultaneously--can throw light on how each one contributes to the relation.

However, the potentials of canonical correlation analysis will only be realized if researchers understand the logic underlying the method and if some serious interpretation pitfalls are avoided. Given a decision that results in hand involve noteworthy effects (based on significance testing, effect sizes, or replicability/invariance analyses--e.g., Thompson, 1989c), most researchers wish to interpret the results to determine which variables in what ways led to observed effects. The purpose of the present work is to explore ways in which researchers can evaluate variable importance in canonical correlation analysis. However, since canonical analysis subsumes other parametric methods as special cases (Bagozzi, 1981; Knapp, 1978; Thompson, 1988a), much of the discussion will generalize to other methods, as will be illustrated here with respect to multiple regression results. First, however, some readers may appreciate a brief review of the basic logic of canonical analysis.

The Basic Logic of Canonical Calculations

Thompson (1984) notes that canonical correlation can be

presented in bivariate terms. This conceptualization is appealing, because most researchers feel very comfortable thinking in terms of the familiar bivariate correlation coefficient. The view is also important because it forces realization that canonical analysis, like all parametric methods, involves the creation of "synthetic" scores for each person. In regression analyses the synthetic scores are the predicted dependent variable scores of each of the subjects, sometimes termed "YHAT"; the bivariate correlation between the subjects' actual dependent variable scores and synthetic dependent variable ("YHAT") scores is the multiple correlation coefficient, while the sum of squares of the "YHAT" scores equals the sum of squares explained. In factor analysis these synthetic variables are the factor scores of each subject on each of the factors. In discriminant analysis these synthetic variables are the discriminant scores of each subject on each of the discriminant functions.

Table 3 presents a small data set that will be employed to illustrate how scores of individuals are converted into the synthetic variables that are actually the focus of a canonical correlation analysis. The 12 cases were randomly sampled from a data base generated in one of the "Heart Smart" studies, an offshoot of the Bogalusa Heart Study longitudinal examination of the origins of cardiovascular disease during childhood. Readers who want to develop a broader context for interpreting these heuristic data can consult "Heart Smart" project descriptions by Downey, Frank, Webber, Harsha, Virgilio, Franklin and Berenson (1987) and

by Downey, Virgilio, Serpas, Nicklas, Arbeit and Berenson (1988); Thompson, Webber and Berenson (1987) represents an example of one of the research reports originating from the project.

INSERT TABLE 3 ABOUT HERE.

The Table 3 data involve two criterion variables, total ("TOTCHOL") and HDL cholesterol ("HDLCHOL")--HDL is the "good" cholesterol. The Table 3 data involve three predictor variables: (a) time in seconds for completing a one mile walk/run ("MILESEC"); (b) the average of six systolic blood pressure measurements taken by a randomly assigned pair of nurses each taking three measurements ("SYSTOLAV"); and (c) ponderosity ("POND"), or weight in kilograms divided by the cubed value of height in meters. Table 3 also reports in parentheses the _Z_-score equivalents of each of the 12 subjects' raw scores on the five variables. Of course, these data are used here only to illustrate the logic of canonical analysis, and the data set is too small to warrant any substantive interpretation. Table 4 presents univariate and bivariate statistics involving these five variables.

INSERT TABLE 4 ABOUT HERE.

Various analytic methods yield weights that are applied to variables to optimize some condition--such weights include beta weights, factor pattern coefficients, and discriminant function coefficients. These weights are all equivalent (e.g., Thompson & Borrello, 1985; Thompson, 1988a), but in canonical correlation

analysis the weights are usually labelled standardized function coefficients. (Historically, the analogous weights employed in classical parametric methods are given different names primarily to confuse students who are trying to understand the methods and their relationships.) These weights are applied to each individual's data to yield the synthetic variables that are the basis for canonical analysis.

However, in canonical analysis several sets of weights and the resulting synthetic variables can be created. These canonical functions are related to factors, are uncorrelated or orthogonal, and can be rotated in various ways (Thompson, 1984; Thorndike, 1976a). The number of functions that can be computed in a canonical analysis equals the number of variables in the smaller of the two variable sets, as explained by Thompson (1984). In the present example, since the smallest variable set consisted of two variables, two canonical functions could be computed. Table 5 presents the canonical function coefficients and other selected results from the analysis produced using the SPSS-X program file presented in Appendix A. Table 5 presents the information in a useful format, since the arrangement of the table entries implies some of the equations used to derive certain results.

INSERT TABLE 5 ABOUT HERE.

Table 6 defines commonly used canonical statistics as simple bivariate correlation coefficients. Table 7 illustrates the computation of the synthetic variables for each of the 12 subjects

using the Function I function coefficients; the reader may wish to compute the corresponding values associated with the Function II results. For a given function, two synthetic scores are produced for each subject--one associated with the composite of weighted criterion variables, and one associated with the composite of weighted predictor variables. For example, as noted in Table 7, the criterion synthetic variable score, "CRITC1", for subject one was 1.11 ([+0.64*(+0.69)] + [+1.17*(+0.57)]). By the same token, the predictor synthetic variable score for subject 12 was -1.17 ([+0.49*(+0.11)] + [+1.30*(-1.03)] + [+0.70*(+0.16)]).

INSERT TABLES 6 AND 7 ABOUT HERE.

The canonical correlation (Rc) is nothing more (or less) than the Pearson product-moment correlation between the two synthetic variable scores of the subjects on a given function. This can be illustrated in several ways using the present results. For example, the bivariate correlation equals the sum of the cross-products of the two variables, the sum then being divided by n - 1. The cross products of the synthetic variables for each of the 12 subjects are presented in Table 7, as is the sum of these cross products. The sum divided by n - 1 (8.435684/11) equals, within rounding error, the actual Rc result (+.767**2 = 58.8%) reported in Table 5 for Function I.

An alternative presentation is graphic. Figure 1 presents the scattergram in which the 12 pairs of synthetic variable scores from Table 7 are arrayed. For example, note that the fourth subject's

composite scores in Table 7 indicate that this subject is represented by the point ("4") in the upper-right position within the scattergram Figure 1 also presents the least squares regression line best fitting these asterisks. In the two variable case, since the synthetic variables have means of zero, the slope of this regression line equals a beta weight, also equals the bivariate correlation between the synthetic variables, also equals the canonical correlation coefficient, i.e., .767.

INSERT FIGURE 1 ABOUT HERE.

Table 8 presents computations that illustrate the meaning of two other canonical results, structure coefficients and index coefficients. Structure coefficients have the same meaning in a canonical analysis as in other analyses, i.e., structure coefficients are bivariate correlation coefficients between a given criterion or predictor variable and the synthetic variable involving the variable set to which the variable belongs. For example, since "ZMILESEC" was a predictor variable, the correlation between "ZMILESEC" and "PREDC1" is the structure coefficient for "ZMILESEC". Note that the sum of the crossproducts of "ZMILESEC" and "PREDC1", labelled "XSTRUC" in Table 8, once divided by $n - 1$, equals within rounding error the structure coefficient for "MILESEC" presented in Table 5. An index coefficient is the correlation coefficient between a variable and the synthetic variable consisting of variables from the variable set to which the variable does <u>not</u> belong. Table 8 illustrates the calculation of

the index coefficient for "ZMILESEC" on Function I. Thompson (1984) discusses the importance of index coefficients in greater detail.

INSERT TABLE 8 ABOUT HERE.

Canonical Correlation Analysis (CCA) as a General Method Subsuming Univariate Methods, Including Multiple Regression

In a seminal article, Cohen (1968, p. 426) noted that ANOVA and ANCOVA are special cases of multiple regression analysis, and argued that in this realization "lie possibilities for more relevant and therefore more powerful exploitation of research data." Since that time researchers have increasingly recognized that conventional multiple regression analysis of data as they were initially collected (no conversion of intervally scaled independent variables into dichotomies or trichotomies) does not discard information or distort reality, and that the general linear model

> ...can be used equally well in experimental or non-experimental research. It can handle continuous and categorical variables. It can handle two, three, four, or more independent variables... Finally, as we will abundantly show, multiple regression analysis can do anything the analysis of variance does--sums of squares, mean squares, F ratios--and more. (Kerlinger & Pedhazur, 1973, p. 3)

Discarding variance is not generally good research practice (Thompson, 1988b). As Kerlinger (1986, p. 558) explains,

> ...partitioning a continuous variable into a

> dichotomy or trichotomy throws information away... To reduce a set of values with a relatively wide range to a dichotomy is to reduce its variance and thus its possible correlation with other variables. A good rule of research data analysis, therefore, is: <u>Do not reduce continuous variables to partitioned variables</u> (dichotomies, trichotomies, etc.) unless compelled to do so by circumstances or the nature of the data (seriously skewed, bimodal, etc.).

Kerlinger (1986, p. 558) notes that variance is the "stuff" on which all analysis is based. Discarding variance by categorizing variables amounts to "squandering of information" (Cohen, 1968, p. 441). As Pedhazur (1982, pp. 452-453) notes,

> Categorization of attribute variables is all too frequently resorted to in the social sciences... It is possible that some of the conflicting evidence in the research literature of a given area may be attributed to the practice of categorization of continuous variables... Categorization leads to a loss of information, and consequently to a less sensitive analysis.

One reason why researchers may be prone to categorizing continuous variables is that some researchers unconsciously and <u>erroneously</u> associate ANOVA with the power of experimental designs. Humphreys (1978, p. 873) notes that:

> The basic fact is that a measure of individual differences is not an independent variable, and it does not become one by categorizing the scores and treating the categories as if they defined a variable under experimental control in a factorially designed analysis of variance.

Similarly, Humphreys and Fleishman (1974, p. 468) note that categorizing variables in a nonexperimental design using an ANOVA analysis "not infrequently produces in both the investigator and his audience the illusion that he has experimental control over the independent variable. Nothing could be more wrong."

As Cliff (1987, p. 130) notes, the practice of discarding variance on intervally scaled predictor variables to perform OVA analyses creates problems in almost all cases:

> Such divisions are not infallible; think of the persons near the borders. Some who should be highs are actually classified as lows, and vice versa. In addition, the "barely highs" are classified the same as the "very highs," even though they are different. Therefore, reducing a reliable variable to a dichotomy makes the variable more unreliable, not less.

These various realizations have led to less frequent use of OVA methods, and to more frequent use of general linear model approaches such as regression (Elmore & Woehlke, 1988; Goodwin & Goodwin, 1985; Willson, 1982). However, canonical correlation

analysis, and not regression analysis, is the most general case of the general linear model (Baggaley, 1981, p. 129; Fornell, 1978, p. 168). In an important article, Knapp (1978, p. 410) demonstrated this in some mathematical detail and concluded that "virtually all of the commonly encountered tests of significance can be treated as special cases of canonical correlation analysis." Thompson (1988a) illustrates how canonical correlation analysis can be employed to implement all the parametric tests that canonical methods subsume as special cases.

Thus, canonical correlation analysis is a powerful analytic paradigm that can be applied to quite a few research problems. The method is valuable because it honors the complexity of reality by simultaneously considering all relationships among variables, and because the analysis does not require that intervally scaled predictor variables be converted to nominal scale.

The linkage c˙ :CA and multiple regression analysis is particularly easy to see, since both procedures are happily explicitly named correlational procedures. However, all classical parametric methods are least squares procedures that implicitly or explicitly (a) use weights, (b) focus on latent synthetic variables, and (c) yield effect sizes analogous to $\underline{r}^2$, i.e., all classical analytic methods are correlational (Knapp, 1978; Thompson, 1988a).

The Table 3 data can be employed to illustrate these linkages in a manner similar to the illustration by Thompson and Borrello (1985). Knapp (1978) provides a more mathematical treatment of the

linkages. Suppose that the researcher wanted to predict "HDLCHOL" with "MILESEC", "SYSTOLAV" and "POND", and did so using both regression and a canonical correlation procedures. When the Appendix B SAS program file was applied to the Table 3 data to yield these analyses, the results presented in Appendix C are part of the printout.

The correlation coefficients are given different names and are printed to a different number of decimal places, but the results are the same. The same identities occur for F and p values, as can be seen by consulting Appendix C. At first pale the weights used in the two analyses are different, but Table 9 illustrates that the weights are merely in a different metric, and can be converted back and forth. Clearly, regression is a special case of canonical correlation analysis, and much of the forthcoming discussion regarding the interpretation of canonical results will generalize to regression situations as well.

INSERT TABLE 9 ABOUT HERE.

Estimating Variable Importance in Canonical Analysis

Canonical correlation analysis is a potent analytic method. But the difficulty of interpreting canonical results can challenge even the most seasoned analyst. As Thompson (1980, pp.1, 16-17) notes, one

> reason why the technique is rarely used involves the difficulties which can be encountered in trying to interpret canonical results... The neophyte student

> of canonical correlation analysis may be overwhelmed by the myriad coefficients which the procedure produces... [But] canonical correlation analysis produces results which can be theoretically rich, and if properly implemented the procedure can adequately capture some of the complex dynamics involved in educational reality.

There are three major analytic choices with respect to evaluating variable importance in canonical analysis: (a) interpretation based on direct examination of sample results in hand <u>at the function level</u> (e.g., function, structure, index and Rc^2 coefficients such as those presented in Table 3); (b) interpretation based on mathematical aggregates of sample coefficients, including results <u>at the model level</u>; and (c) interpretation based on estimates of the generalizability of sample results to other samples of subjects or of variables.

A. <u>Interpretation Based on Sample Results at the Function Level</u>

The two primary rivals for evaluating variable importance at the function level are function coefficients and structure coefficients, though index coefficients can be very useful in certain specialized cases (Thompson, 1984). In terms of actual contemporary analytic practice, Eason, Daniel and Thompson (1990) found that in about one-third of the published canonical studies researchers only report and interpret function coefficients, a situation not unlike that with respect to some researchers' preferences for beta weights in the related regression case.

Some researchers have taken the position that structure coefficients should be emphasized in interpretation, and have grounded their position on the view that structure coefficients should theoretically be more invariant from sample to sample than function coefficients. For example, Darlington, Weinberg & Walberg, 1973, p. 443) argue that:

> However, in most cases the choice between the two is dictated by a practical rather than a theoretical consideration: sampling error. By analogy to the situation in multiple regression (Darlington, 1968, pp. 175-177), it can be inferred that the standard errors of weights [function coefficients] are often much higher than those of correlations [structure coefficients]. This is especially true precisely in those cases when the differences between weights and correlations are greatest--when variables within a set are highly intercorrelated.

Cooley and Lohnes (1971, p. 55) take the same position in the related regression case, and do so on the same grounds. However, Monte Carlo studies (Barcikowski & Stevens, 1975; Thompson, 1989b; Thorndike & Weiss, 1973) have not yet conclusively resolved these issues.

Although these Monte Carlo studies may be criticized on various ground (cf. Thompson, 1989b; Thorndike, 1976b), it appears that neither function nor structure coefficients are more sensitive to sampling error, and that indeed **both function and structure**

coefficients tend to be very unstable from sample to sample. Thus, the value of one set of coefficients versus the other must be determined against the standard of psychometric meaning rather than invariance.

However, on psychometric grounds it can be argued that if one inhabited an <u>artificial</u> world of forced-choices, the analyst might interpret structure coefficients while ignoring function coefficients. Structure coefficients are the most helpful coefficients to consult when interpreting canonical results, although many researchers do not interpret and some do not even report structure coefficients. Since structure coefficients inform the researcher of the correlation between each variable and the synthetic variables, these coefficients are what inform the researcher regarding the meaning of what is actually being correlated in a given analysis. A variable may have a function coefficient of zero but a structure coefficient of one, thus in such a case interpretation based solely on the function coefficient would be seriously misguided.

As noted previously, structure coefficients have the same meaning in the canonical cases as in the other analytic methods that the canonical methods subsume as special cases, i.e., structure coefficients are always correlation coefficients between an observed variable and a latent or synthetic variable. For example, in principal components analysis the correlation between the scores on one variable and the factor scores on one factor is the structure coefficient for that variable on that factor. And as

Gorsuch (1983, p. 207) notes, "the basic matrix for interpreting the factors is the factor structure." Similarly, in a discriminant analysis, the correlation between the scores on a predictor variable and the discriminant function scores on a given function is the structure coefficient for that variable on that function.

In the regression case, the correlation between scores on a predictor variable and the "YHAT" scores is the structure coefficient for the predictor variable. Just as structure coefficients are vitally important in interpreting results in other analytic cases, structure coefficients can be very important in interpreting multiple regression results (Cooley & Lohnes, 1971, pp. 54-55). Thompson and Borrello (1985) present an explanation of this application and an actual research example in which the interpretation solely of beta weights rather than of structure coefficients would conceivably have lead to incorrect conclusions.

Thus, with respect to canonical analysis, Meredith (1964, p. 55) suggested that, "If the variables within each set are moderately intercorrelated the possibility of interpreting the canonical variates by inspection of the appropriate regression weights [function coefficients] is practically nil." Similarly, Kerlinger and Pedhazur (1973, p. 344) argued that, "A canonical correlation analysis also yields weights, which, theoretically at least, are interpreted as regression [beta] weights. These weights [function coefficients] appear to be the weak link in the canonical correlation analysis chain." Levine (1977, p. 20, his emphasis) is even more emphatic:

> I specifically say that one <u>has</u> to do this [interpret structure coefficients] since I firmly believe as long as one wants information about the nature of the canonical correlation relationship, not merely the computation of the [synthetic function] scores, one must have the structure matrix.

The hypothetical results presented in Table 5 were useful in explaining the logic of canonical analysis, but could lead the naive researcher to conclude that function and structure coefficients always yield the same interpretations for a given data set. Such a conclusion would be dangerous! For example, Sexton, McLean, Boyd, Thompson and McCormick (1988) present a canonical analysis in which one variable had a function coefficient of +0.02 on Function I, but the same variable had a structure coefficient of +0.89 on the same function.

In an artificial forced-choice world in which only one coefficient could be consulted, structure coefficients might be preeminent, notwithstanding the views to the contrary (Harris, 1989). But in the <u>real world</u> both coefficients should be reported and consulted in interpretation. <u>Interpretations of canonical results based solely on function coefficients should be eschewed</u>.

However, it should be noted that the function and structure coefficients for a given variable on given functions approach each other as the variables in a variable set approach being uncorrelated (Thompson, 1984, pp. 22-23). In fact, the function and

structure coefficient matrices for a given variable set are identical when the variables in the set are perfectly uncorrelated, as would be the case, for example, if the variables in a set consisted of factor scores on orthogonally rotated principal components.

B. Interpretation Based on Aggregates or at the Model Level

One argument in favor of evaluating variable importance at the function level is that one may only be interested in one function or in fewer than the full set of possible functions. However, good arguments can also be made that interpretation ought to occur at the full model level. For example, in some respects functions must be interpreted in relation to each other, just as orthogonal factors are interpreted in relation to each other. Since canonical functions are perfectly uncorrelated, one can check the meaning assigned to a given function by confirming that the same meaning will not fit any other functions in the model. Furthermore, since the variables were presumably selected based on a theoretical premise that the variables made sense as a set, and since the selection of multivariate methods presumes interest in the full network of variable relationships, it can be argued that evaluation at the model level is more consistent with the analytic choice.

Several aggregates of sample results at a model level can be readily envisioned. One way of aggregating coefficients within the model is used in computing what are called redundancy coefficients (*Rd*). If the squared structure coefficients for a given set of variables on a single function are added and then the sum is

divided by the number of variables in the set, the result informs the researcher regarding how much of the variance in the variables, on the average, is contained within the synthetic scores for that function. This result is called a variate adequacy coefficient (Thompson, 1984). The calculations are illustrated in Table 5.

Stewart and Love (1968) suggested that multiplying the adequacy coefficient times the squared canonical correlation yields a coefficient that they labelled a redundancy coefficient (Rd). Miller (1975) developed a partial test distribution to test the statistical significance of redundancy coefficients. Cooley and Lohnes (1976, p. 212) suggest that redundancy coefficients have great utility. In reality, the interpretation of redundancy coefficients does not make much sense in a conventional canonical analysis, and thus would not be very useful in weighting other results to evaluate a variable's importance.

As Cramer and Nicewander (1979) proved in detail, redundancy coefficients are not truly multivariate (see also Thompson, 1988a). This is very disturbing, because the main argument in favor of multivariate methods (for both substantive and statistical reasons) is that these methods simultaneously consider all relationships during the analysis (Fish, 1988; Thompson, 1986)!

A redundancy coefficient for a given variable set on a given function equals the adequacy coefficient for the set times the squared Rc for the function. The redundancy coefficient can only equal one when the synthetic variables for the function represent all the variance of every variable in the set, and the squared Rc

also exactly equals one. This does not usually occur in practice. Thus, redundancy coefficients are useful only to test outcomes that rarely occur and which may be unexpected (Thompson, 1980, p. 16; Thompson, 1984). Furthermore, it seems contradictory to routinely employ an analysis that uses functions coefficients to optimize *Rc*, and then to interpret results not optimized as part of the analysis, i.e., redundancy coefficients.

However, there are exceptions to most rules. The Sexton et al. (1988) study was a concurrent validity study in which a very large *Rc* was expected on Function I, and in which all variables were expected to be very highly correlated with the synthetic variables defining Function I. In factor analytic language, *G* or *G*eneral structure was expected. In this rather unusual case, the *Rd* coefficient was useful in testing a theoretical expectation. But, again, such results are not usually expected.

But other coefficient aggregates at a model level that may be more useful can be formulated. For example, Rim (1972) suggested that the squared canonical correlation coefficient on a given function might be used to weight other coefficients, and then aggregates of these products might be formed across functions. One particularly intriguing possibility involves the squared structure coefficients of the variables, since squared structure coefficients are pregnant with meaning in the canonical context, and since Rc^2 and squared structure coefficients are both correlation coefficients and are in the same metric.

Consider for illustrative purposes the results for "MILESEC"

presented in Table 3. "MILESEC" shares almost no variance (0.39%) with the function with the largest Rc^2 (58.81%), notwithstanding the fact that the variable has 56.10% of its variance in common with the synthetic variable that yielded a meager Rc^2 of 3.84% on Function II. The proposed weighted aggregate for "MILESEC" would accurately reflect these dynamics (2.38% = .0238 = (.0039 x .5881) + (.5610 x .0384) = .0023 + .0215).

A second intriguing variation invokes a canonical commonality analysis (cf. Thompson, 1988a; Thompson & Miller, 1985). This analysis can be conducted using aggregates constructed at a function level, or by pooling commonality results across functions. Only the construction of aggregates in canonical commonality analysis at a function level will be illustrated here, since the basic logic of canonical commonality analysis can be sufficiently explained with the simpler example. The example employs the Table 3 data and analyses conducted using a portion of the SAS program file presented in Appendix B.

The first step in a canonical commonality analysis is to create the synthetic variable scores for the variable set of primary interest in the research. Presume for the current example that the researcher was interested in predicting scores on both types of cholesterol as a set. Thus, the researcher is willing to consider variations in the predictor variable set. The synthetic criterion variable scores ("CRITC1") for the 12 subjects on Function I are presented in both Tables 7 and 8; Appendices A and B both illustrate their calculation.

The next step in the analysis requires that various combinations of the predictor variables be used to predict the synthetic variable scores using a regression analysis. Table 10 presents these results for the current example. Of course, the $\underline{R}^2$ using all three predictor (.588108) equals the $\underline{R}c^2$ (58.81%) presented in Table 5, and actually is a canonical correlation coefficient.

INSERT TABLE 10 ABOUT HERE.

Finally, the Table 10 results must be manipulated to determine how much explained variance in "CRITC1" is uniquely provided by each predictor, and how much is common to the predictors in various combinations. Mood (1969) presents the algorithms to determine how to compute these estimates, but Cooley and Lohnes (1976, p. 222) have tabled the required computations for cases involving up to four variables. In addition to reporting a regression cancer study that might have been grossly misinterpreted absent a commonality analysis, Seibold and McPhee (1979, p. 358) table the procedures for cases involving up to five variables. Table 11 illustrates the computations for the current example.

INSERT TABLE 11 ABOUT HERE.

Table 12 summarizes the analysis in a manner that helps to clarify what a canonical commonality analysis does--canonical commonality analysis decomposes the squared index coefficients for the variables used to predict a synthetic score composite. In this

approach a variable is deemed important if its accounts for a preponderant portion of the synthetic variable composite, particularly if it does so uniquely. This approach is most informative when the use of index coefficients would also make sense (Thompson, 1984). Such use presumes (a) interest in one variable set taken as a given but (b) willingness to use the variables in the other set in various combinations. This is not sensible if the analyst believes that both variable sets in a given situation must be treated as inviolate wholes in the circumstances at hand. Sometimes canonical commonality analysis will be useful; sometimes it won't.

INSERT TABLE 12 ABOUT HERE.

A third plausible aggregate of coefficients focuses on evaluating variable importance at a full model level. One good candidate for such interpretation is the sum of the squared structure coefficients for a variable, summed across all possible functions, i.e., the canonical communality coefficient ($\underline{h}^2$). Variables with large communality coefficients have variance that was (or could have been) used in forming the synthetic latent variables that are related in the analysis. However, variables can have relatively large communality coefficients and still not be terribly valuable. For example, as noted in Table 3, "MILESEC" has a communality coefficient of 56.49%, but most of this variance arises on Function II with its $\underline{R}c^2$ of .0384. "MILESEC" has a squared structure coefficient of less than one percent (.0039) on

Function I, the function with a squared canonical correlation of 58.81%.

Thus, communality coefficients are primarily useful is evaluating variable importance in an observe sense, i.e., in evaluating which variables are <u>not</u> important in a given model. Variables with extremely low communality coefficients do not add much to the solution on any functions. In fact, it has been suggested that communality coefficients can be used to eliminate variables from an analysis (Thompson, 1982, 1984), though other criteria for isolating less valuable variables have also been proposed (Rim, 1972; Thorndike & Weiss, 1969, 1983). Such logics move toward discussion of the third system for evaluating variable importance.

C. <u>Interpretation Based on Generalizability of Sample Results to Other Samples of Subjects or of Variables</u>

The business of science is formulating **generalizable** insight. No one study, taken singly, establishes the basis for such insight. As Neale and Liebert (1986, p. 290) observe:

> No one study, however shrewdly designed and carefully executed, can provide convincing support for a causal hypothesis or theoretical statement... Too many possible (if not plausible) confounds, limitations on generality, and alternative interpretations can be offered for any one observation. Moreover, each of the basic methods of research (experimental, correlational, and case

> study) and techniques of comparison (within- or between-subjects) has intrinsic limitations. How, then, does social science theory advance through research? The answer is, by collecting a diverse body of evidence about any major theoretical proposition.

Thus, the ultimate test of a variable's importance is how well the variable leads to noteworthy insights that generalize across samples of subjects and samples of variables.

Evaluating the generalizability of canonical results to other samples of subjects or of variables is a daunting task, but a task which the serious scholar can ill-afford to shirk. Prior to presenting several logics for evaluating result generalizability, it may be worthwhile to explore the extent to which canonical correlation analysis capitalizes on sampling error. As Nunnally (1978, p. 298) notes, "one tends to take advantage of chance in any situation [all parametric methods] where something is optimized from the data at hand", as in least squares methods.

For illustrative purposes a population of 10,000 cases of Z-scores with the same correlation matrix as that presented in Table 4 was created using the program written by Morris (1975). Three random samples of n=50 were then drawn from this population. The three samples were essentially independent in terms of overlapping membership; only one case was randomly selected to be in more than one sample, i.e., the case was selected to be in two of the samples. Table 13 presents canonical results associated with both

the full population and the three samples.

INSERT TABLE 13 ABOUT HERE.

Note that the three samples do tend to slightly overestimate population Rc^2 values. However, even when the sample ratio of subjects to variables is as small as 10:1, the capitalization on sampling error is relatively small for both functions. This pattern is generally consistent with reports in previous Monte Carlo work (Thompson, in press).

But coefficients for individual variables tend to be less invariant across samples, though both function and structure coefficients tend to be **equally unstable**. For example, compare the function and structure coefficients for "MILESEC" on Function II in sample two (.565 and .600, respectively) and in sample three (.957 and .977, respectively). The results in the last sample suggest that "MILESEC" is substantially more important with respect to this function, e.g., the squared structure coefficients are 36.0% ($.600^2$) versus 95.4% ($.977^2$). Again, this pattern of greater variability for function and structure coefficients (as against Rc^2) is consistent with previous Monte Carlo work (cf. Thompson, 1989b). One important implication of such findings is that researchers must be very cautious in generalizing about variable importance based on a single sample of data.

A second implication is that formulas that correct sample estimates to support generalization to the full population or to other samples might be useful in constructing the weighted

aggregates discussed previously. For example, one might employ Wherry's (1931) correction formula to $\underline{R}c^2$, as suggested by Cliff (1987, p. 446). Thompson's (in press) empirical results support the use of such a correction. The Wherry correction can be expressed as:

$$R^2 - ((1 - R^2) * (v / (n - v - 1))).$$

When applied to the most inflated $\underline{R}c^2$ result for Function I presented in Table 13, i.e., the result for sample #2, the corrected population estimate is:

```
.6228 - ((1 - .6228) * (5 / (50 - 5 - 1)))
.6228 - (   .3772    * (50 /      44    ))
.6228 - (   .3772    *        .113636    )
.6228 -                 .042863
.579936,
```

a result which slightly overcorrects in estimating the true, known population value of .5927.

However, Stevens (1986, pp. 78-84) incisively implies that researchers usually ground their work in empirical findings from previous samples, and in actual practice usually want their work to generalize to future samples in future research rather than to the unknowable population. Herzberg (1969) provides a correction for this estimate that also might be used in creating coefficient aggregates to evaluate variable importance:

$$1 - ((n-1)/(n-v-1))((n-2)/(n-v-2))((n+1)/n)(1-R^2).$$

For these data the correction for the first $\underline{R}c^2$ would be:

```
1 -  ( 49 / 44    )  ( 48 / 43    )  ( 51/50)(1-.6228)
1 -    1.1136363   *  1.1162790   *  1.02 *  0.3772
1 -          1.2431289            *  1.02 *  0.3772
1 -                          1.2679915    *  0.3772
1 -                                    0.4782864
```

0.5217135,

a result which even further overcorrects the estimate, and is thus still more conservative.

Efforts to estimate the sampling specificity of coefficients for specific variables are more difficult, or at least more tedious. Some researchers randomly split their sample data, conduct separate analyses for the two subgroups, and then subjectively compare the results to determine if they appear to be similar. Two points need to be emphasized about such an approach.

First, such procedures almost always overestimate the invariance or generalizability of results, as Thompson (1984, p. 46) emphasizes. Most researchers work with samples of convenience that are homogeneous in several if not many respects, e.g., geographic location. The members of the random subgroups, then, have more in common with each other than will independent future samples drawn by other researchers. This is not said to discourage the practice of replicability analysis, but is emphasized only to give a context for interpretation of results. It is always better to have an empirical overestimate of result replicability than to have merely a dogmatic attachment to the presumption that sample results will generalize.

Second, it is emphasized that inferences regarding replicability **must** be made **empirically** rather than subjectively, e.g., not by visually comparing coefficients across two randomly identified sample subgroups. Subjective comparisons will not do, because the functions in the two solutions may not occupy a common

factor space. Functions that appear to be quite different may in fact yield quite similar synthetic variable scores--apparent differences in functions yielding comparable values for the synthetic variables actually related in canonical analysis are not very noteworthy (Thompson, 1989c). Cliff (1987, pp. 177-178) suggests that such cases involve "insensitivity" of the weights to departures from least squares constraints.

Three of the many possible logics to empirically estimate which variables yield variance that generalizes as being useful in the canonical solution are discussed here. The first two logics involve randomly splitting the sample into two subgroups (Thompson, 1984, pp. 41-47). Appendix D presents an SPSS-X program file that was applied to the Table 3 data for illustrative purposes.

The first method of evaluating result invariance is cross-validation. In the present example separate analyses were conducted for both the two subgroups each consisting of a different set of six subjects. These results are presented in Table 14. The first technique requires that the weights of each be applied both to the subgroup's own data as well was to the other subgroup's data, as illustrated in Appendix D. The cross-validation is only reported here for the first canonical function, but the same logic could be applied to other functions. The result is eight synthetic variables (e.g., "PREDC00" is the latent variable for the predictor variable set constructed using subgroup zero's data and its own function coefficients; "PREDC01" is the latent variable for the predictor variable set constructed using subgroup zero's data and the

function coefficients from the other subgroup).

INSERT TABLE 14 ABOUT HERE.

These eight variables are then correlated to yield the results presented in Table 15. When results are invariant the difference (called shrinkage) between the "actual" Rc^2's and the "shrunken" Rc^2's will be small. These values for the present example are identified in the footnotes to Table 15. The tabled results indicate that even virtually perfect canonical correlation coefficients can shrink even to virtually zero when sample size is small. Of course, the current heuristic example involves analyses of five variables across only six subjects, and invariance would hardly be expected in such cases.

INSERT TABLE 15 ABOUT HERE.

The second logic compares results at the full model level. In this logic the function coefficients are rotated to "best fit" position so that the functions occupy a common factor space (Thompson, 1984, pp. 43-46). The RELATE program written by Veldman (1967) can be used for this purpose. Table 16 presents the functions coefficients for subgroup one rotated to best fit position with the function coefficients for subgroup zero. The cosines of the angles between the functions across the subgroups ($|.94|$) suggest that the two functions can be successfully rotated to best fit. However, before the analyst can legitimately interpret this result, the cosines of individual variables across the

subgroups must be consulted (cf. Gorsuch, 1983, p. 284). For the current example these values were: -0.0258, 0.9607, 0.8305, -0.5624, and 0.1268. One expects the cosines of the variables to be homogeneous and large, preferably greater than .90. Thus, these coefficients too suggest that the example results are not invariant across samples.

INSERT TABLE 16 ABOUT HERE.

With respect to both these first two logics, it needs to be emphasized that **the results for the full sample are always the best basis for substantive interpretation**, since the full sample involves the largest sample size. That is, the results from the two subgroups are not themselves subjected to any substantive interpretation. The results for subgroups are merely consulted to get some idea of how much confidence to vest in the results for the full sample.

One problem with both the sampling splitting logics is that for any given sample there are usually many possible splits available, and different splits may yield different and even contradictory results for the same data sample. A more sophisticated logic relies on the bootstrap approach popularized by Efron (1979; Diaconis & Efron, 1983; Lunneborg, in press). In this approach what is conceptually done is to copy the data into a "megaset" over and over again many times. Then one draws numerous random samples (often 500 or 1,000) from the "megaset" and pools (averages) results across the samples. Since the different samples

each involve different configurations of subjects, the analyst is evaluating the degree to which idiosyncratic characteristics of a few subjects may limit the generalizability of the results to future samples.

Lunneborg (1987) presents some excellent computer programs that can be used on a PC to conduct analyses for many research situations. However, this application in the canonical case involves some special problems. Results from each random sample **must** be rotated to best fit position with some target solution **before** results are pooled across samples. If this is not done, unbeknownst to the researcher a function may emerge as Function I in sample one but as Function II in another sample, and the researcher will be averaging apples and oranges and will reach unnecessarily dire predictions regarding the sampling specificity of results. The target solution can be defined in several ways, e.g., a theoretically expected matrix (usually consisting of +1's, 0's, and -1's) or the actual results in hand can be used as the target solution. Such an approach could be developed by generalizing the routines presented in the related factor analytic case by Thompson (1988c).

Another approach involves bootstrap estimation of the population bivariate correlation matrix. If this matrix turns out so that the correlation submatrix for the variables in the predictor set and the correlation submatrix for the criterion variable set can both be "inverted" (see Thompson, 1984, p. 13), then the bootstrap estimated correlation matrix might be subjected

to canonical analysis in the stead of the sample *r* matrix in hand. Table 17 presents such bootstrap estimates of the population correlation matrix derived from 500 random samples from the conceptual "megaset" for the Table 3 data.

INSERT TABLE 17 ABOUT HERE.

The standard deviations about mean estimates in bootstrap applications are especially useful. These are sophisticated estimates of the standard errors of results, informed by the data in hand rather than by paltry assumptions about the likely distribution of sample-estimates of parameters. Thus, Lunneborg (1987, p. 38, his emphasis) characterizes these as "*real*" estimates.

Of course, in most studies researchers wish to generalize to other samples of variables also measuring specified constructs, and not only to other samples of subjects. Thus, many researchers find it useful to conduct leave-one-out (L-O-O) analyses for different subsets of variables. When variables that are most important to a given solution are removed, the coefficients (including Rc^2) will change appreciably. The backward variable elimination strategies discussed previously can be characterized as falling with this genre, though the methods focusing on communality coefficients do not require supplementary analyses. These various L-O-O strategies become especially potent when they are themselves used in conjunction with replicability analysis procedures, e.g., cross-validation.

Summary

The paper has explained the basic logic of canonical correlation analysis. It was noted that all statistical tests implicitly involve the calculation of least squares weights, and that all parametric tests can be conducted using canonical analysis, since canonical analysis subsumes parametric methods as special cases. Canonical analysis is potent because it does not require the researcher to discard variance of any of the variables, and because the analysis honors the complexity of a reality in which variables interact simultaneously.

Three major classes of procedures for evaluating the importance of specific variables in canonical correlation analysis were explored. Various procedures in each class were illustrated in a concrete fashion using a single small data set for heuristic purposes. Appended SPSS-X and SAS program files may facilitate further exploration of the concepts argued here.

Krus, Reynolds and Krus (1976, p. 725) argue that, "Dormant for nearly half a century, Hotelling's (1935, 1936) canonical variate analysis has come of age. The principal reason behind its resurrection was its computerization and inclusion in major statistical packages." The potentials of canonical analyses will be fully realized when this sophisticated method is understood and at least some of the interpretation aids identified herein are thoughtfully invoked.

References

Baggaley, A. R. (1981). Multivariate analysis: An introduction for consumers of behavioral research. Evaluation Review, 5, 123-131.

Bagozzi, R.P. (1981). Canonical correlation analysis as a special case of a structural relations model. Multivariate Behavioral Research, 16, 437-454.

Barcikowski, R.S., & Stevens, J.P. (1975). A Monte Carlo study of the stability of canonical correlations, canonical weights and canonical variate-variable correlations. Multivariate Behavioral Research, 10, 353-364.

Bray, J.H., & Maxwell, S.E. (1982). Analyzing and interpreting significant MANOVAs. Review of Educational Research, 52, 340-367.

Bray, J.H., & Maxwell, S.E. (1985). Multivariate analysis of variance. Newbury Park: SAGE.

Cliff, N. (1987). Analyzing multivariate data. San Diego: Harcourt Brace Jovanovich.

Cohen, J. (1968). Multiple regression as a general data-analytic system. Psychological Bulletin, 70, 426-443.

Cooley, W. W., & Lohnes, P. R. (1971). Multivariate data analysis. New York: John Wiley and Sons.

Cooley, W. W., & Lohnes, P. R. (1976). Evaluation research in education. New York: Irvington.

Cramer, E. M., & Nicewander, W. A. (1979). Some symmetric, invariant measures of multivariate association. Psychometrika, 44, 43-54.

Darlington, R.B., Weinberg, S.L., & Walberg, H.J. (1973). Canonical variate analysis and related techniques. Review of Educational Research, 43, 433-454.

Diaconis, P., & Efron, B. (1983). Computer-intensive methods in statistics. Scientific American, 248(5), 116-130.

Downey, A.M., Frank, G.C., Webber, L.S., Harsha, D.W., Virgilio, S., Franklin, F.A., & Berenson, G.S. (1987). Implementation of "Heart Smart": A cardiovascular school health promotion program. Journal of School Health, 57, 98-104.

Downey, A.M., Virgilio, S., Serpas, D.C., Nicklas, T.A., Arbeit, M.L., & Berenson, G.S. (1988). "Heart Smart"--A staff development model for a school-based cardiovascular health intervention. Health Education, 19(5), 64-71.

Eason, S., Daniel, L., & Thompson, B. (1990, January). A review of practice in a decade's worth of canonical correlation analysis studies. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin, TX.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. The Annals of Statistics, 7, 1-26.

Elmore, P.B., & Woehlke, P.L. (1988). Statistical methods employed in American Educational Research Journal, Educational Researcher, and Review of Educational Research from 1978 to 1987. Educational Researcher, 17(9), 19-20.

Fish, L.J. (1988). Why multivariate methods are usually vital. Measurement and Evaluation in Counseling and Development, 21, 130-137.

Fornell, C. (1978). Three approaches to canonical analysis. Journal of the Market Research Society, 20, 166-181.

Goodwin, L.D., & Goodwin, W.L. (1985). Statistical techniques in AERJ articles, 1979-1983: The preparation of graduate students to read the educational research literature. Educational Researcher, 14(2), 5-11.

Gorsuch, R. L. (1983). Factor analysis (2nd ed.). Hillsdale, NJ: Erlbaum.

Harı R.J. (1989). A canonical cautionary. Multivariate Behavioral Research, 24, 17-39.

Herzberg, P.A. (1969). The parameters of cross validation. Psychometrika, Monograph supplement, No. 16.

Hinkle, D.E., Wiersma, W., & Jurs, S.G. (1979). Applied statistics for the behavioral sciences. Chicago: Rand McNally.

Hotelling, H. (1935). The most predictable criterion. Journal of Experimental Psychology, 26, 139-142.

Huberty, C.J. (1987). On statistical testing. Educational Researcher, 16(8), 4-9.

Huberty, C.J, & Morris, J.D. (1989). Multivariate analysis versus multiple univariate analysis. Psychological Bulletin, 105, 302-308.

Hummel, T.J., & Johnston, C.B. (1986, April). An empirical comparison of size and power of seven methods for analyzing multivariate data in the two-sample case. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service

No. ED 269 480)

Humphreys, L.G. (1978). Doing research the hard way: Substituting analysis of variance for a problem in correlational analysis. Journal of Educational Psychology, 70, 873-876.

Humphreys, L.G., & Fleishman, A. (1974). Pseudo-orthogonal and other analysis of variance designs involving individual-differences variables. Journal of Educational Psychology, 66, 464-472.

Kerlinger, F. N. (1973). Foundations of behavioral research (2nd ed.). New York: Holt, Rinehart and Winston.

Kerlinger, F. N. (1986). Foundations of behavioral research (3rd ed.). New York: Holt, Rinehart and Winston.

Kerlinger, F. N., & Pedhazur, E. J. (1973). Multiple regression in behavioral research. New York: Holt, Rinehart and Winston.

Knapp, T. R. (1978). Canonical correlation analysis: A general parametric significance testing system. Psychological Bulletin, 85, 410-416.

Krus, D.J., Reynolds, T.S., & Krus, P.H. (1976). Rotation in canonical variate analysis. Educational and Psychological Measurement, 36, 725-730.

Levine, M. S. (1977). Canonical analysis and factor comparison. Beverly Hills: SAGE.

Lunneborg, C.E. (1987). Bootstrap applications for the behavioral sciences. Seattle: University of Washington.

Lunneborg, C.E. (in press). [Review of Computer intensive methods for testing hypotheses]. Educational and Psychological

Measurement.

Maxwell, S. (in press). Recent developments in MANOVA applications. In B. Thompson (Ed.), Advances in social science methodology (Vol. 2). Greenwich, CT: JAI Press.

Meredith, W. (1964). Canonical correlations with fallible data. Psychometrika, 29, 55-65.

Miller, J. K. (1975). The sampling distribution and a test for the significance of the bimultivariate redundancy statistic: A Monte Carlo Study. Multivariate Behavior Research, 10, 233-244.

Mood, A.M. (1969). Macro-analysis of the American educational system. Operations Research, 17, 770-784.

Morris, J.D. (1975). A computer program to create a population with any desired centroid and covariance matrix. Educational and Psychological Measurement, 35, 707-710.

Morrow, J.R., Jr., & Frankiewicz, R.G. (1979). Strategies for the analysis of repeated and multiple measures designs. Research Quarterly, 50, 297-304.

Neale, J.M., & Liebert, R.M. (1986). Science and behavior: An introduction to methods of research (3rd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Nunnally, J.C. (1978). Psychometric theory (2nd ed.). New York: McGraw-Hill.

Pedhazur, E. J. (1982). Multiple regression in behavioral research: Explanation and prediction (2nd ed.). New York: Holt, Rinehart and Winston.

Rim, E. (1972). A stepwise canonical approach to the selection of

"kernel" variables from two sets of variables (Doctoral dissertation, University of Illinois at Urbana, 1972). Dissertation Abstracts International, 34, 623A. (University Microfilms No. 73-17,386)

Seibold, D.R., & McPhee, R.D. (1979). Commonality analysis: A method for decomposing explained variance in multiple regression analyses. Human Communication Research, 5, 355-365.

Sexton, J.D., McLean, M., Boyd, R.D., Thompson, B., & McCormick, K. (1988). Criterion-related validity of a new standardized developmental measure for use with infants who are handicapped. Measurement and Evaluation in Counseling and Development, 21, 16-24.

Stewart, D. K., & Love, W. A. (1968). A general canonical correlation index. Psychological Bulletin, 70, 160-163.

Stevens, J. (1986). Applied multivariate statistics for the social sciences. Hillsdale, NJ: Erlbaum.

Tatsuoka, M. M. (1973). Multivariate analysis in educational research. In F. N. Kerlinger (Ed.), Review of research in education (pp. 273-319). Itasca, IL: Peacock.

Thompson, B. (1980, April). Canonical correlation: Recent extensions for modelling educational processes. Paper presented at the annual meeting of the American Educational Research Association, Boston. (ERIC Document Reproduction Service No. ED 199 269)

Thompson, B. (1982, February). Stepwise canonical correlation analysis. Paper presented at the annual meeting of the Southwest

Educational Research Association, Austin, TX. (ERIC Document Reproduction Service No. ED 222 546)

Thompson, B. (1984). Canonical correlation analysis: Uses and interpretation. Newbury Park: SAGE.

Thompson, B. (1986, November). Two reasons why multivariate methods are usually vital. Paper presented at the annual meeting of the Mid-South Educational Research Association, Memphis.

Thompson, B. (1988a, April). Canonical correlation analysis: An explanation with comments on correct practice. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED 295 957)

Thompson, B. (1988b). Discarding variance: A cardinal sin in research. Measurement and Evaluation in Counseling and Development, 21, 3-4.

Thompson, B. (1988c). Program FACSTRAP: A program that computes bootstrap estimates of factor structure. Educational and Psychological Measurement, 48, 681-686.

Thompson, B. (1989a, August). Applications of multivariate statistics: A bibliography of canonical correlation analysis studies. Paper presented that the annual meeting of the American Psychological Association, New Orleans. (ERIC Reproduction Service No. forthcoming)

Thompson, B. (1989b, March). Invariance of multivariate results: A Monte Carlo study of canonical coefficients. Paper presented at the annual meeting of the American Educational Research

Association, San Francisco. (ERIC Document Reproduction Service No. ED 306 226)

Thompson, B. (1989c). Statistical significance, result importance, and result generalizability: Three noteworthy but somewhat different issues. Measurement and Evaluation in Counseling and Development, 22, 2-6.

Thompson, B. (in press). Finding a correction for the sampling error in multivariate measures of relationship: A Monte Carlo study. Educational and Psychological Measurement.

Thompson, B., & Borrello, G. M. (1985). The importance of structure coefficients in regression research. Educational and Psychological Measurement, 45, 203-209.

Thompson, B., & Miller, J.H. (1985, February). A multivariate method of commonality analysis. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin. (ERIC Document Reproduction Service No. ED 263 151)

Thompson, B., Webber, L., & Berenson, G. S. (1987). Factor structure of a children's health locus of control measure: A confirmatory maximum-likelihood analysis. Educational and Psychological Measurement, 47, 1071-1080.

Thorndike, R. M. (1976a, April). Strategies for rotating canonical components. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 123 259)

Thorndike, R.M. (1976b). Studying canonical analysis: Comments on Barcikowski and Stevens. Multivariate Behavioral Research, 11,

249-253.

Thorndike, R.M., & Weiss, D.J. (1969, May). An empirical investigation of stepwise canonical correlation. Paper presented at the annual meeting of the Midwestern Psychological Association, Chicago.

Thorndike, R.M., & Weiss, D.J. (1973). A study of the stability of canonical correlations and canonical components. Educational and Psychological Research, 33, 123-184.

Thorndike, R.M., & Weiss, D.J. (1983). An empirical investigation of step-down canonical correlation analysis with cross-validation. Multivariate Behavioral Research, 18, 183-196.

Veldman, D. (1967). FORTRAN programming for the behavioral sciences. New York: Holt, Rinehart and Winston.

Wherry, R.J. (1931). A new formula for predicting the shrinkage of the coefficient of multiple correlation. Annals of Mathematical Statistics, 2, 440-451.

Willson, V. L. (1982, January). Misuses of regression approaches to ANOVA and ANCOVA. Paper presented at the annual meeting of the Southwest Educational Research Association, Austin.

Witte, R.S. (1985). Statistics (2nd ed.). New York: Holt, Rinehart and Winston.

Wood, D.A., & Erskine, J.A. (1976). Strategies in canonical correlation with application to behavioral data. Educational and Psychological Measurement, 36, 861-878.

Table 1
All Possible Families of Outcomes
for a Fair Coin Flipped Three Times

	Flip # 1	2	3
1.	T	T	T
2.	H	T	T
3.	T	H	T
4.	T	T	H
5.	H	H	T
6.	H	T	H
7.	T	H	H
8.	H	H	H
p of H on each Flip	50%	50%	50%

p of 1 **or more** H's (TW error analog) in set of 3 Flips = 7/8 = 87.5%

or

where TW error analog = .50,
EW $p = 1 - (1 - .5)^3$
$= 1 - .5^3 = 1 - .125 = .875$

Note. The probability of one _or more_ occurrences of a given outcome in a set of events is $1 - (1-p)^k$, where _p_ is the probability of the given occurrence on each trial and _k_ is the number of trials in a set of perfectly independent events.

Table 2
Formula for Estimating Experimentwise Type I Error Inflation
When Hypotheses are Perfectly Uncorrelated

	TW alpha		Tests		Experimentwise alpha
1 - (1 -	0.05)	**	1	=	
1 - (	0.95)	**	1	=	
1 -	0.95			=	0.05000[a]
Range Over Testwise (TW) alpha = .01					
1 - (1 -	0.01)	**	5	=	0.04901
1 - (1 -	0.01)	**	10	=	0.09562
1 - (1 -	0.01)	**	20	=	0.18209
Range Over Testwise (TW) alpha = .05					
1 - (1 -	0.05)	**	5	=	0.22622
1 - (1 -	0.05)	**	10	=	0.40126
1 - (1 -	0.05)	**	20	=	0.64151
Range Over Testwise (TW) alpha = .10					
1 - (1 -	0.10)	**	5	=	0.40951
1 - (1 -	0.10)	**	10	=	0.65132
1 - (1 -	0.10)	**	20	=	0.87842

Note. "**" = "raise to the power of".

[a]These calculations are presented (a) to illustrate the implementation of the formula step by step and (b) to demonstrate that when only one test is conducted, the experimentwise error rate equals the testwise error rate, as should be expected if the formula behaves properly.

Table 3
Data Set for Heuristic Example

n	MILESEC	SYSTOLAV	POND	TOTCHOL	HDLCHOL
1	890(+0.18)	94.0(-1.04)	11.5(-0.91)	180(+0.64)	80.1(+1.17)
2	1097(+1.16)	108.7(+1.42)	12.0(-0.69)	142(-1.56)	51.1(-1.02)
3	1300(+2.12)	97.7(-0.42)	13.1(-0.21)	165(-0.23)	63.3(-0.10)
4	948(+0.45)	90.3(-1.66)	12.6(-0.43)	199(+1.74)	75.7(+0.84)
5	940(+0.41)	100.7(+0.08)	19.3(+2.49)	187(+1.04)	61.0(-0.27)
6	760(-0.44)	104.3(+0.69)	14.7(+0.48)	148(-1.22)	76.0(+0.86)
7	740(-0.53)	95.3(-0.82)	14.2(+0.26)	164(-0.29)	78.5(+1.05)
8	571(-1.33)	97.7(-0.42)	13.6(+0.00)	174(+0.29)	54.3(-0.78)
9	748(-0.50)	102.7(+0.42)	10.9(-1.17)	190(+1.22)	62.2(-0.18)
10	640(-1.01)	96.0(-0.70)	11.4(-0.95)	161(-0.46)	67.4(+0.21)
11	642(-1.00)	107.0(+1.14)	14.6(+0.44)	159(-0.58)	34.8(-2.25)
12	957(+0.49)	108.0(+1.30)	15.2(+0.70)	159(-0.58)	70.8(+0.47)

Table 4
Descriptive Statistics and Correlation Coefficients

	MILESEC	SYSTOLAV	POND	TOTCHOL	HDLCHOL	PREDC1	CRITC1
Mean	852.8	100.2	13.6	169.0	64.6	0.0	0.0
SD	211.0	6.0	2.3	17.3	13.2	1.0	1.0
MILESEC		.052	.046	-.047	.140	.063	.048
SYSTOLAV			.244	-.624	-.559	-.981	-.752
POND				.008	-.121	-.084	-.064
TOTCHOL					.243	.637	.830
HDLCHOL						.569	.742
PREDC1							.767

Note. The bivariate correlation (.767) between PREDC1 and CRITC1 is also called *Rc*.

Table 5
Canonical Coefficients for Table 3 Data

	Function I				Function II				
	Func.	Index	Struc.	Squared Struc.	Func.	Index	Struc.	Squared Struc.	h^2
MILESEC	.108	.048	.063	.39%	-.818	-.156	-.749	56.10%	56.49%
SYSTOLAV	-1.026	-.753	-.981	96.28%	-.106	.001	.005	.00%	96.28%
POND	.161	-.064	-.084	.70%	.625	.110	.562	31.57%	32.27%
Adequacy				32.46%				29.22%	
Redundancy				19.09%				1.12%	
R^2				58.81%				3.84%	
Redundancy				36.47%				1.46%	
Adequacy				62.01%				37.99%	
TOTCHOL	.691	.637	.830	68.95%	.765	.109	.557	31.05%	100.00%
HDLCHOL	.574	.569	.742	55.07%	-.856	-.131	-.670	44.93%	100.00%

Table 6
Bivariate Equivalents of Canonical Coefficients

Variable	Type	Variable	Type	Result
CRITC1	Latent	PREDC1	Latent	Function I Rc
CRITC2	Latent	PREDC2	Latent	Function II Rc
CRITC1	Latent	CRITC2	Latent	r = 0
CRITC1	Latent	PREDC2	Latent	r = 0
TOTCHOL	Observed	CRITC1	Latent	Structure Coef. for TOTCHOL on Function I
HDLCHOL	Observed	CRITC1	Latent	Structure Coef. for HDLCHOL on Function I
MILESEC	Observed	PREDC1	Latent	Structure Coef. for MILESEC on Function I
SYSTOLAV	Observed	PREDC1	Latent	Structure Coef. for SYSTOLAV on Function I
TOTCHOL	Observed	PREDC1	Latent	Index Coef. for TOTCHOL on Function I
HDLCHOL	Observed	PREDC1	Latent	Index Coef. for HDLCHOL on Function I
MILESEC	Observed	CRITC1	Latent	Index Coef. for MILESEC on Function I
SYSTOLAV	Observed	CRITC1	Latent	Index Coef. for SYSTOLAV on Function I

Table 7
"Synthetic" Variate Scores for Function I

ZMILESEC	ZSYSTOLA	ZPOND	ZTOTCHOL	ZHDLCHOL	PREDC1	CRITC1	PRxCR
0.18	-1.04	-0.91	0.64	1.17	0.94	1.11	1.04
1.16	1.42	-0.69	-1.56	-1.02	-1.45	-1.66	2.41
2.12	-0.42	-0.21	-0.23	-0.10	0.62	-0.22	-0.13
0.45	-1.66	-0.43	1.74	0.84	1.68	1.68	2.82
0.41	0.08	2.49	1.04	-0.27	0.36	0.56	0.20
-0.44	0.69	0.48	-1.22	0.86	-0.67	-0.35	0.23
-0.53	-0.82	0.27	-0.29	1.05	0.83	0.40	0.33
-1.33	-0.42	0.00	0.29	-0.78	0.29	-0.25	-0.07
-0.50	0.42	-1.17	1.22	-0.18	-0.67	0.74	-0.49
-1.01	-0.70	-0.95	-0.46	0.21	0.46	-0.20	-0.09
-1.00	1.14	0.44	-0.58	-2.25	-1.20	-1.69	2.04
0.49	1.30	0.70	-0.58	0.47	-1.17	-0.13	0.15
							8.435684

Note. The sum of the cross-products (8.435684) divided by n-1 (11) is, within rounding error, the canonical correlation, i.e., .767.

Table 8
Calculation of Structure and Index Coefficients

ZMILESEC	PREDC1	CRITC1	XSTRUC	XINDEX
0.18	0.94	1.11	0.17	0.20
1.16	-1.45	-1.66	-1.67	-1.93
2.12	0.62	-0.22	1.32	-0.46
0.45	1.68	1.68	0.76	0.76
0.41	0.36	0.56	0.15	0.23
-0.44	-0.67	-0.35	0.30	0.15
-0.53	0.83	0.40	-0.44	-0.22
-1.33	0.29	-0.25	-0.38	0.33
-0.50	-0.67	0.74	0.33	-0.37
-1.01	0.46	-0.20	-0.46	0.20
-1.00	-1.20	-1.69	1.20	1.69
0.49	-1.17	-0.13	-0.58	-0.06
Sum			0.689188	0.528548

<u>Note</u>. The sum of the cross-products of "ZMILESEC" and "PREDC1" (0.689188) divided by n-1 (11) is +.062653, within rounding error, the structure coefficient of "ZMILESEC" on Function I. The sum of the cross-products of "ZMILESEC" and "CRITC1" (0.528548) divided by n-1 (11) is +.048049, within rounding error, the index coefficient of "ZMILESEC" on Function I.

Table 9
Canonical and Regression Linkages

Variable	Canonical Function Coefficients (F)		Regression beta Weights		r with DV	Structure Coefficients
MILESEC	0.2891	x R =	0.16885304	/ R = F	0.1399	0.2395
SYSTOLAV	-0.9760	x R =	-0.570042	/ R = F	-0.5590	-0.9571
POND	0.0164	x R =	0.009583643	/ R = F	-0.1214	- .2078
Rc	0.584047	Mult R Sq	0.3411			
		(R =	0.5840)			

Note. These results are isolated from Appendix C--the printout from applying Appendix B's SAS program to the Table 3 data. The beta weight for MILESEC (.16885304) equals, within rounding error, the function coefficient for the variable multiplied by _Rc_ (or _R_), i.e., 0.1688479 = .2891 x .584047. The function coefficient for MILESEC (.2891) equals, within rounding error, the beta weight for the variable divided by _R_ (or _Rc_), i.e., 0.2891086505 = .16885304 / .584047. The structure coefficients for the variables are the same in both analyses, though with most computer package regression programs the analyst must compute the structure coefficients by hand. In the regression case, as explained by Thompson and Borrello (1985), a variable's structure coefficient equals the _r_ between the predictor and the dependent variable divided by _R_, e.g., .2395 = .1399 / .584047.

Table 10
Rc^2 for all Combinations of Predictors Used to Predict PREDC1

Coef. #	Squared Rc	Combination
1	0.002308	MILESEC (1)
2	0.566202	SYSTOLAV (2)
3	0.004119	POND (3)
4	0.573766	MILESEC SYSTOLAV
5	0.006727	MILESEC POND
6	0.581260	SYSTOLAV POND
7	0.588108	MILESEC SYSTOLAV POND

Table 11
Variance Partitions for Unique and Common Variance

U(1) =	R7	- R6		
0.006847=	0.588108	- 0.581260		
U(2) =	R7	- R5		
0.581381=	0.588108	- 0.006727		
U(3) =	R7	- R4		
0.014342=	0.588108	- 0.573766		
C(1,2) =	R5	+ R6	- R3	- R7
-0.00423=	0.006727	+ 0.581260	- 0.004119	- 0.588108
C(1,3) =	R4	+ R6	- R2	- R7
0.000716=	0.573766	+ 0.581260	- 0.566202	- 0.588108
C(2,3) =	R4	+ R5	- R1	- R7
-0.00992=	0.573766	+ 0.006727	- 0.002308	- 0.588108
C(1,2,3)=	R1	+ R2	+ R3	+ R7
-0.00101=	0.002308	+ 0.566202	+ 0.004119	+ 0.588108
	- R4	- R5	- R6	
	0.573766	- 0 006727	- 0.581260	

Table 12
Canonical Commonality Summary Table

	MILESEC	SYSTOLAV	POND
Unique to MILESEC (1)	0.006847		
Unique to SYSTOLAV (2)		0.581381	
Unique to POND (3)			0.014342
Common to (1,2)	-0.00423	-0.00423	
Common to (1,3)	0.000716		0.000716
Common to (2,3)		-0.00992	-0.00992
Common to (1,2,3)	-0.00101	-0.00101	-0.00101
Sum of Partitions	0.002308	0.566202	0.004119
Table 5 Index Coef.	0.048	-0.753	-0.064
Squared Index Coef.	0.002304	0.567009	0.004096

Table 13
Canonical Results for Modelled Population (N=10,000)
and Three Random Samples (n=50) from the Population

N=10,000	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	0.109	0.064	0.40%	-0.821	-0.797	63.57%	63.98%
SYSTOLAV	-1.026	-0.981	96.31%	-0.109	0.001	0.00%	96.31%
POND	0.159	-0.088	0.77%	0.621	0.556	30.94%	31.71%
Adequacy			32.50%			31.51%	
Rd			19.26%			1.23%	
R^2			59.27%			3.89%	
Rd			36.89%			1.47%	
Adequacy			62.23%			37.77%	
TOTCHOL	0.688	0.830	68.88%	0.769	0.558	31.13%	100.00%
HDLCHOL	0.576	0.746	55.59%	-0.856	-0.666	44.41%	100.00%

n=50 #1	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	0.018	0.223	4.97%	0.937	0.887	78.61%	83.58%
SYSTOLAV	-0.968	-0.984	96.84%	0.101	-0.055	0.30%	97.14%
POND	0.177	0.245	6.00%	-0.447	-0.392	15.36%	21.35%
Adequacy			35.93%			31.42%	
Rd			22.20%			1.51%	
R^2			61.78%			4.79%	
Rd			39.07%			1.76%	
Adequacy			63.24%			36.76%	
TOTCHOL	0.715	0.859	73.83%	-0.754	-0.512	26.17%	100.00%
HDLCHOL	0.532	0.726	52.64%	0.893	0.688	47.36%	100.00%

n=50 #2	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	0.219	0.346	11.99%	0.565	0.600	35.98%	47.96%
SYSTOLAV	-0.[illegible]88	-0.960	92.22%	0.311	-0.027	0.07%	92.29%
POND	0.198	-0.123	1.50%	-0.838	-0.799	63.80%	65.31%
Adequacy			35.24%			33.28%	
Rd			21.94%			1.04%	
R^2			62.28%			3.12%	
Rd			38.17%			1.21%	
Adequacy			61.29%			38.71%	
TOTCHOL	0.431	0.633	40.01%	-0.939	-0.775	59.99%	100.00%
HDLCHOL	0.800	0.909	82.57%	0.654	0.417	17.43%	100.00%

Table 13 (continued)

n=50 #3	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	0.260	0.120	1.44%	0.957	0.977	95.50%	96.95%
SYSTOLAV	-1.017	-0.961	92.35%	0.106	0.272	7.38%	99.73%
POND	0.102	-0.081	0.66%	0.166	0.218	4.76%	5.42%
Adequacy			31.48%			35.88%	
Rd			18.03%			1.63%	
R^2			57.26%			4.55%	
Rd			36.83%			1.62%	
Adequacy			64.33%			35.67%	
TOTCHOL	0.512	0.725	52.61%	-0.913	-0.688	47.39%	100.00%
HDLCHOL	0.721	0.872	76.04%	0.759	0.489	23.96%	100.00%

Note. On two of the three samples the second function was computed with opposite signs than occurred in the population or the third sample. However, the signs of a function are arbitrary--one can always reverse a function by multiplying all the function and structure coefficients for the function by -1.

Table 14
Canonical Solutions Across Two Subgroups (n_1=6 + n_2=6 = 12)

Sample #1

	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	0.325	0.388	15.09%	0.832	0.643	41.32%	56.41%
SYSTOLAV	0.942	0.922	85.07%	-0.321	-0.138	1.90%	86.97%
POND	-0.148	-0.034	0.11%	0.788	0.535	28.59%	28.70%
Adequacy			33.43%			23.94%	
Rd			33.38%			13.60%	
R^2			99.86%			56.84%	
Rd			71.33%			16.24%	
Adequacy			71.43%			28.57%	
TOTCHOL	-0.666	-0.888	78.82%	0.888	0.460	21.18%	100.00%
HDLCHOL	-0.511	-0.800	64.03%	-0.986	-0.600	35.97%	100.00%

Sample #2

	Func.	Struc.	Squared Struc.	Func.	Struc.	Squared Struc.	h^2
MILESEC	1.059	0.508	25.77%	0.000	-0.225	5.06%	30.83%
SYSTOLAV	-0.996	-0.457	20.90%	0.268	-0.179	3.20%	24.09%
POND	-0.062	-0.113	1.28%	-1.081	-0.970	94.04%	95.33%
Adequacy			15.98%			34.10%	
Rd			15.94%			15.30%	
R^2			99.72%			44.86%	
Rd			49.85%			22.44%	
Adequacy			49.99%			50.01%	
TOTCHOL	-0.017	0.004	0.00%	1.000	1.000	100.00%	100.00%
HDLCHOL	1.000	1.000	99.97%	-0.004	0.017	0.03%	100.00%

Table 15
"Actual" and "Shrunken" Rc^2 Values
as Bivariate Correlation Coefficients (Appendix D)

	PREDC00	PREDC01	PREDC11	PREDC10
CRITC00	.9993[a]	-.3477	.	.
CRITC01	-.7904	.0255[b]	.	.
CRITC11	.	.	.9986[c]	-.2273
CRITC10	.	.	-.6049	.1854[d]

Note. There are no cases for correlations for which only a decimal is presented.

[a]the "actual" Rc for invariance group 0 subjects
[b]the "shrunken" Rc for invariance group 0 subjects when group 1's weights are used
[c]the "actual" Rc for invariance group 1 subjects
[d]the "shrunken" Rc for invariance group 1 subjects when group 0's weights are used

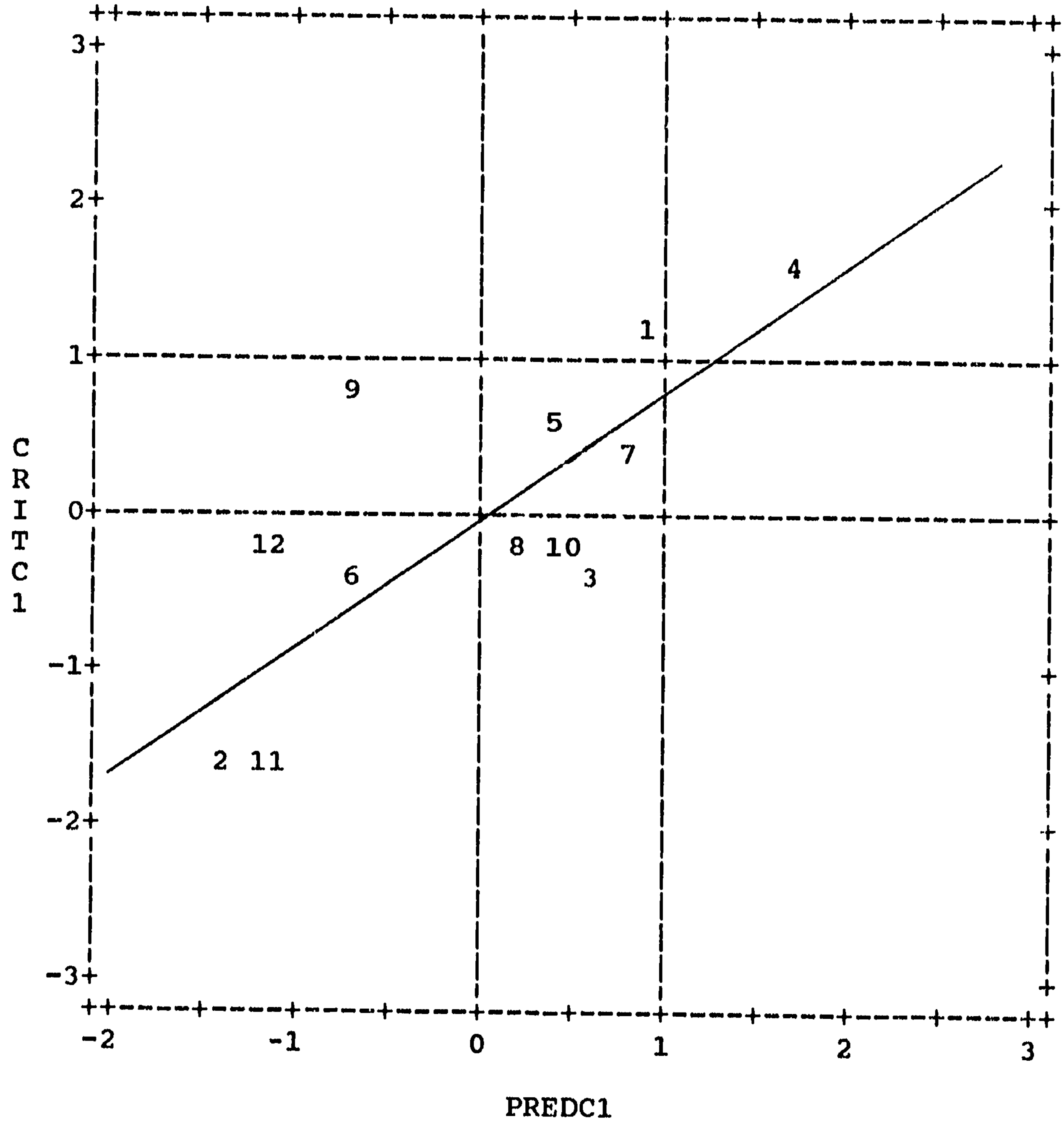
Table 16
Function Matrix for Subgroup 1 Rotated to Best Fit with Function Matrix for Subgroup 2

NEW B:	1	2
1	-0.9961	0.3603
2	0.8451	-0.5908
3	0.4262	0.9950
4	-0.3245	-0.9461
5	-0.9392	0.3439

Table 17
Bootstrap Estimates of r's for Table 3 Data

Coef.	Table 4 Estimate	Mean	Median	SD
1	0.052	0.0514	0.0417	0.2819
2	0.046	0.0421	0.0603	0.2287
3	-0.047	-0.0233	-0.0489	0.2690
4	0.140	0.1135	0.1551	0.3092
5	0.244	0.2598	0.2428	0.2343
6	-0.624	-0.5878	-0.6196	0.2135
7	-0.559	-0.5430	-0.5737	0.2166
8	0.008	-0.0649	-0.0519	0.3541
9	-0.121	-0.0971	-0.1198	0.2224
10	0.243	0.2189	0.2486	0.2560

Figure 1
Scattergram of Latent Composite Scores on Function I



Note. Each subject is represented within the scattergram by a case sequence number.

Appendix A:
SSPS-X Program File Used to Compute Reported Results

```
TITLE 'RANDOM SAMPLE OF n=12 OF N=367 "HEART SMART" ##'
FILE HANDLE BT/NAME='CANONHPE.DTA'
DATA LIST FILE=BT/OBS 1-3 SEX 10 RACE 12 HEIGHT 18-23 (2)
  WEIGHT 25-29 (2) MILESEC 5-8 SYSTOLAV 31-35 (1) POND 45-48 (1)
  TOTCHOL 14-16 HDLCHOL 37-40 (1)
LIST VARIABLES=ALL/CASES=12/FORMAT=NUMBERED
CORRELATIONS VARIABLES=MILESEC TO HDLCHOL
MANOVA MILESEC TO POND WITH TOTCHOL HDLCHOL/
  PRINT=SIGNIF(DIMENR EIGEN) DISCRIM(STAN COR ALPHA(1.00))/DESIGN
descriptives variables=milesec to hdlchol/statistics=all/save
compute predc1=(.10812*zmilesec)+(-1.02600*zsystola)+(.16110*zpond)
compute critc1=(.69095*ztotchol)+(.57439*zhdlchol)
compute predc2=(-.81751*zmilesec)+(-.10555*zsystola)+(.62539*zpond)
compute critc2=(.76497*ztotchol)+(-.85597*zhdlchol)
print formats zmilesec to critc2 (F9.5)
list variables=milesec zmilesec systolav zsystola pond zpond
  totchol ztotchol hdlchol zhdlchol predc1 critc1 predc2 critc2/
  cases=12/format=numbered
correlations variables=milesec to hdlchol predc1 critc1
  predc2 critc2/statistics=descriptives
plot title='Scattergram of Latent Composite Scores on Function I'/
  vertical=reference(0,1) min(-3) max(5)/
  horizontal=reference(0,1) min(-3) max(4)/
  plot=critc1 with predc1
```

Note. Command lines in upper case are required for usual canonical analyses. The commands in lower case were used for heuristic purposes, i.e., to make explicit the logic of canonical analysis.

Appendix B:
SAS Program File Used to Demonstrate CCA to R Linkages

```
TITLE 'CCA AND R LINKAGE';
DATA CCA; INFILE CANONHPE;
INPUT OBS 1-3 SEX 10 RACE 12 HEIGHT 18-23 WEIGHT 25-29
  MILESEC 5-8 SYSTOLAV 31-35 POND 45-48 TOTCHOL 14-16 HDLCHOL 37-40;
PROC PRINT; VAR OBS MILESEC SYSTOLAV POND TOTCHOL HDLCHOL;
TITLE 'CCA SUBSUMES R';
PROC REG; MODEL HDLCHOL=MILESEC SYSTOLAV POND/STB;
PROC CANCORR ALL; VAR HDLCHOL; WITH MILESEC SYSTOLAV POND;
TITLE 'CCA COMMONALITY ANALYSIS';
PROC CANCORR ALL; VAR TOTCHOL HDLCHOL; WITH MILESEC SYSTOLAV POND;
DATA CCANEW; SET CCA; ZTOTCHOL=(TOTCHOL-169.)/17.2837286;
ZHDLCHOL=(HDLCHOL-64.6)/13.2448961;
 CRITC1=(.691*ZTOTCHOL)+(.5744*ZHDLCHOL);
PROC PRINT;
PROC RSQUARE; MODEL CRITC1=MILESEC SYSTOLAV POND;
```

Note. SAS is more useful in demonstrating CCA linkages, because it follows a multivariate approach if the user requests one, even if the problem really defaults to a simpler model. SPSS used to do so, but in the more recent version will change procedures to the simplest model possible.

The SAS RSQUARE procedure is particularly useful when conducting a commonality analysis to partition variance, since the procedure computes results for all possible combinations of predictors.

Appendix C:
Abridged (but Unchanged) Printout from Applying the Appendix B Program File to the Table 3 Data

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	3	658.24063413	219.41354471	1.381	0.3170
ERROR	8	1271.45937	158.93242073		
C TOTAL	11	1929.70000			

ROOT MSE	12.60684	R-SQUARE	0.3411
DEP MEAN	64.6	ADJ R-SQ	0.0940
C.V.	19.51523		

VARIABLE	DF	STANDARDIZED ESTIMATE
INTERCEP	1	0
MILESEC	1	0.16885304
SYSTOLAV	1	-0.570042
POND	1	0.009583643

CORRELATIONS AMONG THE 'WITH' VARIABLES

	MILESEC	SYSTOLAV	POND
MILESEC	1.0000	0.0516	0.0463
SYSTOLAV	0.0516	1.0000	0.2435
POND	0.0463	0.2435	1.0000

CORRELATIONS BETWEEN THE 'VAR' VARIABLES AND THE 'WITH' VARIABLES

	MILESEC	SYSTOLAV	POND
HDLCHOL	0.1399	-0.5590	-0.1214

CANONICAL CORRELATION	ADJUSTED CANONICAL CORRELATION	APPROX STANDARD ERROR	SQUARED CANONICAL CORRELATION
0.584047	0.498980	0.198663	0.341110

LIKELIHOOD RATIO	TESTS OF H0: F	NUM DF	DEN DF	PR > F
0.65888965	1.3805	3	8	0.3170

STANDARDIZED CANONICAL COEFFICIENTS FOR THE 'WITH' VARIABLES

	W1
MILESEC	0.2891
SYSTOLAV	-0.9760
POND	0.0164

CORRELATIONS BETWEEN THE 'WITH' VARIABLES AND THEIR CANONICAL VARIABLES

	W1
MILESEC	0.2395
SYSTOLAV	-0.9571
POND	-0.2078

Appendix D:
SPSS-X Program File to Conduct Cross-Validation Invariance Study

```
TITLE 'RANDOM SAMPLE OF n=12 OF N=367  "HEART SMART" ##'
FILE HANDLE BT/NAME='CANONHPE.DTA'
DATA LIST FILE=BT/OBS 1-3 SEX 10 RACE 12 HEIGHT 18-23 (2) WEIGHT 25-29 (2)
  MILESEC 5-8 SYSTOLAV 31-35 (1) POND 45-48 (1) TOTCHOL 14-16 HDLCHOL 37-40 (1)
LIST VARIABLES=ALL/CASES=12/FORMAT=NUMBERED
CORRELATIONS VARIABLES=MILESEC TO HDLCHOL
MANOVA MILESEC TO POND WITH TOTCHOL HDLCHOL/PRINT=SIGNIF(DIMENR
  EIGEN) DISCRIM(STAN COR ALPHA(1.00))/DESIGN
compute inv=1
if (obs lt 7)inv=0
comment *****in real research invariance groups RANDOMLY created
subtitle 'invariance group 0   n=1 to 6'
temporary
select if (inv eq 0)
descriptives variables=milesec to hdlchol
if (inv eq 0)zmilese0=(milesec-989.167)/186.927
if (inv eq 0)zsystol0=(systolav-99.283)/6.737
if (inv eq 0)zpond0=(pond-13.867)/2.881
if (inv eq 0)ztotcho0=(totchol-170.167)/22.463
if (inv eq 0)zhdlcho0=(hdlchol-67.867)/11.192
temporary
select if (inv eq 0)
manova milesec to pond with totchol hdlchol/print=signif(dimenr
  eigen) discrim(stan cor alpha(1.00))/design
subtitle 'invariance group 1   n=7 to 12'
temporary
select if (inv eq 1)
descriptives variables=milesec to hdlchol
if (inv eq 1)zmilese1=(milesec-716.333)/135.615
if (inv eq 1)zsystol1=(systolav-101.117)/5.589
if (inv eq 1)zpond1=(pond-13.317)/1.765
if (inv eq 1)ztotchol=(totchol-167.833)/12.222
if (inv eq 1)zhdlchol=(hdlchol-61.333)/15.332
temporary
select if (inv eq 1)
manova milesec to pond with totchol hdlchol/print=signif(dimenr
  eigen) discrim(stan cor alpha(1.00))/design
descriptives variables=zmilese0 to zhdlchol/
subtitle 'true Rc and cross-validation Rc results'
compute predc00=(.32523*zmilese0)+(.94178*zsystol0)+(-.14830*zpond0)
compute predc01=(1.05927*zmilese0)+(-.99572*zsystol0)+(-.06219*zpond0)
compute predc11=(1.05927*zmilese1)+(-.99572*zsystol1)+(-.06219*zpond1)
compute predc10=(.32523*zmilese1)+(.94178*zsystol1)+(-.14830*zpond1)
compute critc00=(-.66584*ztotcho0)+(-.51092*zhdlcho0)
compute critc01=(-.01683*ztotcho0)+(1.00021*zhdlcho0)
compute critc11=(-.01683*ztotchol)+(1.00021*zhdlchol)
compute critc10=(-.66584*ztotchol)+(-.51092*zhdlchol)
variable labels predc00 'grp 0 data with grp 0 weights'
                predc01 'grp 0 data with grp 1 weights'
```

```
               predc11 'grp 1 data with grp 1 weights'
               predc10 'grp 1 data with grp 0 weights'
               critc00 'grp 0 data with grp 0 weights'
               critc01 'grp 0 data with grp 1 weights'
               critc11 'grp 1 data with grp 1 weights'
               critc10 'grp 1 data with grp 0 weights'
print formats zmilese0 to critc10 (f5.2)
list variables=obs zmilese0 to critc10/cases=12
correlations variables=predc00 to critc10/
  statistics=descriptives
```

Note. The analysis requires two runs. The first uses only the content not in bold. This printout is used to find the coefficients needed to add the bolded content used in the second run. In general it is best to not split the sample into exactly equal groups, as was done here (so that there would be sufficient degrees of freedom to run the small example); using groups of slightly different sizes facilitates printout interpretation.